© The Journal of Risk and Insurance 1997 Vol. 64 No. 2 323–345

Risks in Derivatives Markets: Implications for the Insurance Industry

Ludger Hentschel
Clifford W. Smith, Jr.

ABSTRACT

We model default risk in derivative contracts. Firms are less likely to default on derivatives than on corporate bonds because bonds are always liabilities, while derivatives can be assets. We provide an upper bound for default risk in derivatives—one substantially lower than appears to be implied by public debate over derivatives. Systemic risk is the aggregation of default risks; since default risk has been exaggerated, so has systemic risk. Finally, public debate over derivative risks seems to ignore what we call "agency risk." Without careful monitoring, standard compensation plans for derivatives traders can induce traders to take inappropriately risky positions.

INTRODUCTION

The continuing discussion of risks and regulation in derivative markets illustrates that there is little agreement on what the risks are or whether regulation is a useful tool for their control.[1] One source of confusion is the sheer profusion of names describing the risks arising from derivatives. Besides the "price risk" of potential losses on derivatives from changes in interest rates, foreign exchange rates, or commodity prices, there is "default risk" (sometimes referred to as "counterparty risk"), "liquidity (or funding) risk," "legal risk," "settlement risk" (or, a variation thereof, "Herstatt risk"), and "operations risk." Last, but not least, is "systemic risk"—the notion of problems in derivatives markets spreading throughout the financial system that seems to be at the heart of many regulatory concerns.

Ludger Hentschel is Assistant Professor and Clifford W. Smith, Jr., is Clarey Professor of Finance, both in the William E. Simon Graduate School of Business Administration at the University of Rochester, Rochester, New York.

This article has benefited from suggestions by D. H. Chew, G. W. Fenn, S. P. Kothari, R. J. Mackay, C. W. Smithson, N. Valerio, and two anonymous referees. The authors gratefully acknowledge financial support from the John M. Olin Foundation and the Bradley Policy Research Center. Parts of the analysis in this article appear in less technical form in Hentschel and Smith (1995).

[1] We use the term derivative to refer to financial contracts that explicitly have the features of options, futures, forwards, or swaps.

This article analyzes the risks associated with derivative transactions and the impact of regulation in limiting these risks. We provide a simple, parametric framework in which one can analyze price, default, and systemic risk. A deeper understanding of these risks is important for firms in the insurance industry for at least three reasons: (1) insurance companies use derivatives in their risk management activities, (2) the value and risk of the assets in insurance companies' investment portfolios depend on the risk management activities of the firms represented in their portfolios, and (3) some insurance companies are market makers in derivatives.

The next section reviews price risk—that is, the potential for losses on derivative positions stemming from changes in the prices of the underlying assets (for instance, interest rates, exchange rates, and commodity prices). Then, we examine default risk by either party to a derivatives contract—a risk that we believe has been largely misunderstood and exaggerated. The existence of price risk has been documented by several large, highly publicized derivatives losses. There are, however, few examples of default in derivative markets. We argue that this is a trend that can be expected to continue.

Further, we argue that systemic risk is simply the aggregation of default risks faced by individual firms in using derivatives. In brief, we argue that the possibility of widespread default throughout the financial system caused by derivatives has been exaggerated, principally due to the failure to appreciate the low default risk associated with individual derivative contracts. Partly for this reason, current regulatory proposals should be viewed with some skepticism. In particular, none of the proposals recognize the fundamental dependance of default risk on how derivatives are used.

Finally, we suggest that recent, highly publicized losses can be attributed largely to improper compensation, control, and supervision within firms. We define derivative risks stemming from these sources as "agency risks," a reference to the principal-agent conflicts from which they arise. For instance, we believe that some standard evaluation and compensation systems are ill-suited for employees granted decision rights over derivatives transactions. Firms that pay large bonuses based on short-term performance can encourage excessive risk-taking by employees. Despite limited public discussion, it appears that firms are aware of these issues and are working to control the problems.

PRICE RISK

The modern analysis of financial derivatives is unified by the successful application of arbitrage-pricing arguments. In their seminal work, Black and Scholes (1973) and Merton (1973) first showed how to price options—the last class of derivatives to elude this analysis—via absence of arbitrage.

The ability to use arbitrage pricing in valuing derivatives has at least one profound implication for the current public debate on derivatives: Redundant securities logically cannot introduce new, fundamentally different risks into the financial system. To the extent that derivatives are redundant, they cannot increase the aggregate level of risk in the economy. Derivatives can, however, isolate and concentrate existing risks, thereby facilitating their efficient transfer. Indeed, it is precisely this ability to isolate quite specific risks at low transactions costs that makes derivatives such useful risk-management tools.

For derivatives, the composition of the replicating portfolios can vary considerably over time; maintaining these portfolios can involve extensive and costly trading.[2] Even if such trading costs introduce a degree of imprecision into derivative pricing models, virtually all derivatives can be valued using arbitrage methods. (In fact, these transaction costs introduce a degree of imprecision into derivatives pricing models that provides an important reason why derivatives offer more efficient hedges than "synthetic" derivatives used to hedge the same risks.)

The standard use of derivatives is in managing price risks through hedging. Firms with a core business exposure to underlying factors, such as commodity prices, exchange rates, or interest rates, can reduce their net exposures to these factors by assuming offsetting exposures through derivatives. Rational, value-maximizing motivations for such corporate hedging activities are provided by Mayers and Smith (1982, 1987); Main (1983); Stulz (1984); Smith and Stulz (1985); and Froot, Scharfstein, and Stein (1993); among others.

Although risk aversion can provide powerful incentives to hedge for individuals, this usually is not the motive for large public companies whose owners can adjust their risk exposures by adjusting the composition of their portfolios. Rather, current theory suggests that incentives to hedge stem from progressive taxes, contracting costs, or underinvestment problems. All of these issues are internal to the firm and cannot be resolved by external investors. For instance, an insurance firm may be able to reduce its risk of insolvency and the probability of default on policies by hedging its interest rate exposures. If this risk reduction allows the insurance firm to charge higher premiums, for example, then hedging provides benefits that cannot be obtained independently by investors in the insurance company.

Exposure

For simplicity, we assume that the present value of the firm, V_0, is inherently quadratic in a set of future, random payoffs, x_t, which are (not necessarily observable) asset prices,

$$V_0 = x_t\beta - \frac{\theta}{2}\, \beta'\, x_t'\, x_t\beta \qquad (1)$$

for some positive θ. To focus attention on the use of financial derivatives, we assume that the firm is operating with fixed production technology and scale embodied in β; x_t is assumed to have a mean of zero.[3] In this case, the firm has an incen-

---

[2] The trading required to replicate the payoffs depends critically on the other outstanding positions managed by the firm. Required trading costs for a market maker with an extensive book of derivatives positions are generally dramatically less than the sum of the trades to replicate the individual contracts.

[3] Although this gives rise to negative firm values, adding back a constant mean does not produce additional insights.

tive to reduce its exposure to the underlying factors in order to reduce the quadratic cost term.

This formulation can be interpreted as a local approximation to firm value for any of the aforementioned hedging motives. For example, if taxes motivate hedging, after-tax income is a concave function of income, and the quadratic specification can be viewed as an approximation to after-tax income. Similarly, if insolvency or underinvestment costs rise as firm value falls, then this formulation can approximate such behavior.[4]

Hedging

Next, assume that there is a set of financial instruments with mean zero (net) payoffs $\mathbf{p}_t$. We interpret these instruments as derivatives, but the formulation can be applied to any financial contract in efficient markets. For derivative contracts such as forwards, futures, or swaps, whose payoffs are linear in the underlying prices, the payoffs $\mathbf{p}_t$ are perfectly correlated with the underlying prices.[5]

If the firm can transact in this financial market without costs, then firm value is given by

$$V_H = \mathbf{x}_t\beta + \mathbf{p}_t\gamma - \frac{\theta}{2}\,(\mathbf{x}_t\beta + \mathbf{p}_t\gamma)'\,(\mathbf{x}_t\beta + \mathbf{p}_t\gamma), \tag{2}$$

when the firm holds γ_i units of the ith financial instrument with payoff $p_{i,t}$.

For derivatives such as forwards and options, the single payment date is an accurate representation of the actual contract. For derivatives such as futures and swaps with multiple payment dates, this characterization ignores the sequential nature of payments. (For these contracts, the single payment date can be interpreted as the maturity date of the contract, with all payments cumulated to maturity.)

The characterization of the firm and its derivatives positions in equation (2) abstracts from most dynamic intertemporal features. The single-period framework, however, permits us to focus more clearly on the relation between derivatives positions and default than alternative dynamic approaches. Johnson and Stulz (1987), for example, assume that the net worth of an option writer and the underlying price have a constant correlation that is exogenous to their model. Conditional on this assumption, they can exactly price options with default risk. Cooper and Mello (1991), Sorensen and Bollier (1994), Jarrow and Turnbull (1995), and Longstaff and Schwartz (1995) also provide pricing models for financial contracts in the presence of default risk. They make the same tradeoff as Johnson and Stulz

---

[4] Although we do not view risk aversion as the typical hedging motivation for firms, our parameterization of firm value also can be interpreted as a quadratic approximation to the concave utility function of a risk-averse owner. This interpretation may, however, be relevant for mutual insurers and their policyholders/owners.

[5] Referring to these cases, we will not always make a careful distinction between underlying prices and payoffs. For contracts such as options, whose payoffs are nonlinear in the underlying price, this distinction is important, however.

(1987) and take the source of the default risk as exogenous to their model. In contrast, we structure our model specifically to illustrate that the correlation between firm value and derivative obligations is a crucial ingredient in default risk, and furthermore that this correlation depends on the firm's derivatives position.

Impact of Hedging

The optimal position in the financial contracts is found by optimizing the value of the firm, V_H, over γ, which is equivalent to

$$\min_{\gamma} E\,(x_t\beta + p_t\gamma)'\,(x_t\beta + p_t\gamma) \tag{3}$$

given β. The optimal hedge position is given by

$$\gamma^* = -\Sigma_{pp}^{-1}\,\Sigma_{px}\,\beta = -\Sigma_{pp}^{-1}\,\mathrm{Cov}\,(p_t, V_0), \tag{4}$$

where Σ_{pp} is the variance matrix of p_t, and $\Sigma_{px} = E(p_t'x_t)$ is the matrix of covariances between the elements of p_t and x_t.

In the simple case of a scalar p_t, the optimal hedge position can be written in more familiar form as

$$\gamma^* = -\frac{\mathrm{Cov}(p_t, V_0)}{\mathrm{Var}(p_t)} = -\rho\,\frac{\sigma_{V_0}}{\sigma_p}, \tag{5}$$

where ρ is the correlation between p_t and unhedged firm value, V_0.

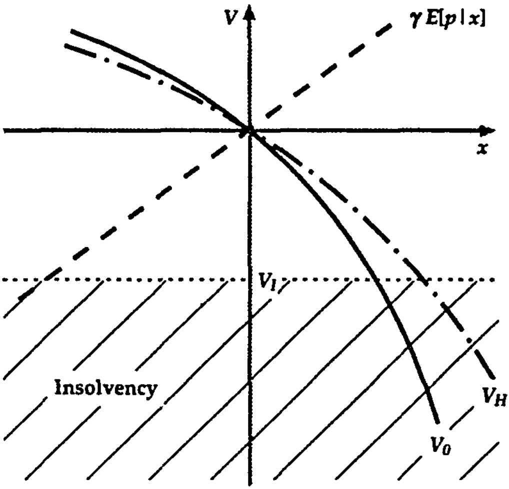
To be an effective hedge, the financial contract must be highly correlated with firm value. To illustrate, if ρ is close to zero, the optimal hedge position is small in order to limit the introduction of additional uncertainty through variations in p_t when the hedging benefits derived from offsetting variations in x_t and p_t are small. Indeed, one of the key risk management advantages of derivatives is that they individually target potentially large sources of variation, such as interest rates, exchange rates, or commodity prices without introducing additional sources of variation.

Figure 1 presents these results graphically for a univariate x_t. The underlying core exposure to x_t is shown by the solid curve labeled V_0, which assumes that the firm's exposure to x is negative.[6] To display the payoff from the financial hedge on the two-dimensional graph, we project it onto x as $\gamma E[p|x]$, shown by the dashed line. Finally, the dashed and dotted curve labeled V_H shows the net exposure of firm value including the derivatives contracts.

---

[6] If x_t is symmetrically distributed around zero, the linear regression of V_0 onto x_t produces a slope coefficient of β. Adler and Dumas (1984) and Flannery and James (1984) interpret β as exposure in the context of exchange and interest rates, respectively.

Figure 1
Firm Value, Exposures, and Hedging



As we suggest in the introduction, insurance companies should be interested in the impact of using derivatives to hedge both because they directly hedge their own exposures and because the insurance companies' loan customers use derivatives to manage their exposures.

Figure 1 can be applied to an insurance firm with an interest rate exposure if we interpret x_t as the interest rate. If the effective maturity of a life insurer's assets differs from that of its liabilities, then the insurer is exposed to unexpected changes in interest rates. For example, with longer duration assets than liabilities, increases in interest rates reduce firm value. This situation gives rise to the downward-sloping value function shown in Figure 1.[7]

The ability of the insurer to meet its financial commitments under its contracts is a primary concern of both current and potential customers. As the insurer's value approaches the level that would make the firm insolvent, V_I, customers are less and less willing to pay premiums, thereby leading to more rapid declines in firm value. This situation gives rise to the concave value function shown in Figure 1.

---

[7] Conversely, with shorter duration assets than liabilities, increases in interest rates would reduce firm value. In our framework, the value of such a firm would be described by a diagram like that in Figure 1, except that V_0 would have a positive slope. That is to say V_0 would be reflected about the vertical axis in Figure 1. Although, for an insurer with shorter maturity assets than liabilities, V_0 would have a positive slope, it still would be concave.

Hedging operates through two channels. First, the hedge reduces the net exposure to the underlying risk factor, which reduces variation in firm value. This is evident in Figure 1 from the reduced slope of V_H compared to V_0. Second and more important, V_H is less concave than V_0. This increases the expected firm value for a given variability of x_t. For an insurer, reducing the probability of financial distress means that the firm can raise the premiums customers are willing to pay.

Note that this reduction in concavity can be achieved with derivatives that have linear payoffs. However, options are derivatives that have nonlinear payoffs that can be used to construct a hedge portfolio with an even more concentrated effect on this aspect of the problem.

Insurers historically have embedded implicit interest rate options in both their assets and liabilities. For instance, whole-life policies frequently contain policy-loan options providing a guaranteed line of credit at a fixed, or capped, rate of interest and options to purchase additional insurance at guaranteed rates. Moreover, single-premium guaranteed investment contracts contain prepayment or redemption provisions. These embedded options are more valuable if interest rates rise.

Since the life insurance company has written options to both insurance and loan customers, value changes produced by interest rate changes are not symmetric. Increases in interest rates lead to a reduction in loan repayments while policyholders reduce their demand for optional fixed-price insurance but increase their demand for fixed-rate policy loans. These embedded financial options cause the gains from declines in interest rates to be less than the losses associated with rate increases. As a result, value is reduced by increases in interest rate volatility. To hedge this risk, insurers can buy interest rate options.

When considering assets in the insurance firm's investment portfolio, one can apply Figure 1 to the insurer's loan customers. In this context, V might be the value of a chemical producer and x_t the price of crude oil. As the price of oil—an input—rises, the chemical producer's value declines, and so does its ability to make loan payments. Ulitmately, the firm becomes insolvent. If, however, the chemical producer hedges its oil exposures, oil price changes represent a smaller risk to lenders. This reduction in the risk of insolvency can be seen in Figure 1, where the hedged chemical producer can withstand larger increases in oil prices before becoming insolvent.

Default Risk on Derivatives

Default risk is the risk that losses will be incurred due to default by the counterparty. As noted above, part of the confusion in the current debate about derivatives stems from the profusion of names associated with default risk. Terms such as "credit risk" and "counterparty risk" are essentially synonyms for default risk. "Legal risk" refers to the enforceability of the contract. Terms such as "settlement risk" and "Herstatt risk" refer to defaults that occur at a specific point in the life of the contract: the date of settlement. These terms do not represent independent risks; they just describe different occasions or causes of default.

In many derivatives contracts, either party may default during the life of the contract.[8] In a swap, for example, either side could default on any of the settlement dates during the life of the swap. In practice, a firm may be able to accelerate default. For example, once it becomes clear that a firm will ultimately be unable to meet all of its obligations, the firm may elect to enter bankruptcy proceedings now, even though current obligations do not force this step. However, the firm would only choose this path if it is in the firm's best interest, and hence there generally will be an optimal default policy. Although such timing issues may be important, especially for firms near bankruptcy, we abstract from these complications and continue to focus on the default risk posed by one side of the contract at a single payment date.[9]

Default risk has two components: the expected exposure (the expected replacement cost of the contract minus the expected recovery from the counterparty) and the probability that default will occur.

Expected Exposure

Expected exposure measures how much capital is likely to be at risk should the counterparty default. The notional principal of derivatives like forwards and swaps grossly overstates actual default exposure. For example, in interest rate swaps, only net interest payments are exchanged. These payments are a small fraction of the notional principal of the swap. In fact, the U.S. General Accounting Office estimates that the net credit exposure on swaps is approximately 1 percent of notional principal.

In addition to the expected value of the derivative at the time of default, expected exposure also depends on expected recovery rates after default. (Current capital standards implicitly assume that the recovery rate is zero, which leads to a material overstatement of the expected loss.) Most swaps are unsecured claims in bankruptcy proceedings. For unsecured financial claims, recovery rates average about 50 percent; for collateralized claims, recovery rates are closer to 80 percent (Franks and Torous, 1994).

Finally, the expected exposure depends on whether the contract includes imbedded options. Specifically, if the swap stipulates a floor rate, the buyer's obligations and the magnitude of the losses it could cause in a default are contractually limited.

---

[8] Options form the notable exception to this rule. An option buyer cannot default after the purchase of the option since the buyer has no further obligations to pay under the option contract. Hence, in option markets, only the option writer poses default risk.

[9] One reason we believe that this is a reasonable tradeoff is that it is unlikely that payments obligated under derivatives trigger default. The more likely it is that default is triggered by obligated payments to other claimholders, the less important it is to focus on the intertemporal details of the cash flows under derivatives contracts without modeling these details of bond, employee, and lease contracts.

Probability of Default

Default on any financial contract, including derivatives, occurs when two conditions are met simultaneously: a party to the contract owes a payment under the contract, and the counterparty cannot obtain timely payment.[10] Under U.S. law, this means that the defaulting party either has insufficient assets to cover the required payments or has successfully filed for protection under the bankruptcy code.

The fact that default only occurs when two conditions hold simultaneously implies that it is a bivariate phenomenon. To capture the nature of default risk, we therefore must consider the bivariate distribution of the counterparty's obligation under the derivative contract as well as its ability to pay. For simplicity, we assume that the firm's ability to pay is fully described by firm value, V_H. Conversely, we assume that the firm's obligation under the derivative contract is described by the net payoff, γp_t.

By assumption, the firm has insufficient assets when firm value, V_H, falls below some critical level, V_I, at which the firm becomes insolvent. Similarly, the firm owes payments on its derivative contracts when $p_t\gamma < 0$. This situation is illustrated in Figure 2. Although the firm is insolvent in the shaded areas of quadrants II and III, it only owes payments on the derivative in the shaded areas of quadrants III and IV. Hence, default on the derivative is confined to the cross-hatched area in quadrant III.[11]

The probability of default is given by the joint probability that the firm value is below V_I, *and* that the derivative requires payments, which occurs for $p_t\gamma < 0$. Hence, the probability of default equals

$$P(D) = P(V_H < V_I \text{ and } p_t\gamma < 0), \tag{6}$$

which is given by the cumulative density function

$$P(D) \equiv F_t(V_I, 0)$$

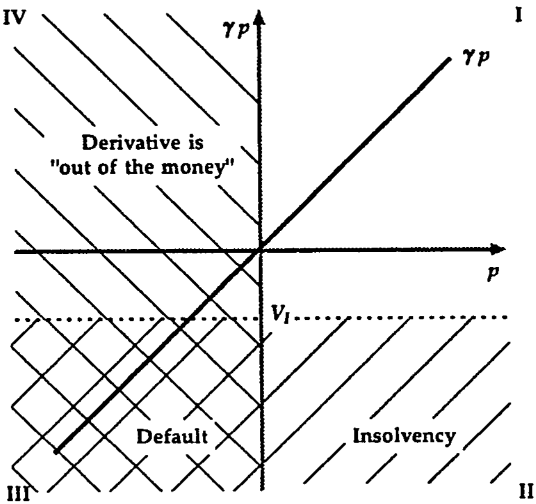
$$= \int_{-\infty}^{V_I} \int \cdots \int_{\{p: p\gamma < 0\}} \left| \prod_i \frac{1}{\gamma_i} \right| f(V_H, p_t)\, dp_1 \ldots dp_n\, dV, \tag{7}$$

---

[10] Our discussion of default generally ignores technical default since it has no direct cash flow consequences. However, many derivative contracts have "cross-default" clauses that can place a party into technical default. Should the counterparty try to unwind the contract under the default terms but fail, then default occurs. On the other hand, if the contract can be unwound at market value, then technical default has no valuation consequences.

[11] Figure 2 reflects currently used swap documentation. A previous standard documentation provided that only the nondefaulting party was entitled to receive a payment upon an event of default (such as filing for bankruptcy). This earlier form would imply that, upon insolvency, the value of the swap would be the min {γp, 0}. Such a change would affect the value of the swap but not our calculations of expected default costs.

Figure 2
Insolvency and Default on Derivatives



where $f(V_H, p_t)$ is the joint probability density of firm value and derivative payoffs For scalar p_t and positive γ, the cumulative density can be found from

$$P(D) = \int_{-\infty}^{V_I} \int_{-\infty}^{0} \frac{1}{\gamma} f(V_H, p_t)\, dp\, dV. \tag{8}$$

Moreover, if we are willing to make specific distributional assumptions about $f(V_H, p_t)$, then we can evaluate default risk quantitatively. Alternatively, one can make distributional assumptions for the underlying prices, x_t and p_t, and aggregate the densities to $f(V_H, p_t)$. (If x_t and p_t are jointly Gaussian, for example, the density $f(V_H, p_t)$ would be a mixture of multivariate Gaussian and Wishart densities.) In principle, one could also directly estimate the joint density function $f(V_H, p_t)$ via standard parametric or nonparametric methods. As we will argue later, however, default is a relatively rare phenomenon, which implies that the default probability depends strongly on the tail behavior of the density. This makes precise empirical estimates difficult.

Complete hedging. Figure 3 illustrates a contour plot of the probability density $f(V_H, p_t)$ and indicates the combinations of V_H and γp_t for which the firm will be insolvent and in default, respectively. In Figure 3, the inside contour contains the combinations of γp_t and V_H that will occur with a probability of 95 percent. Consequently, there is only a 5 percent probability that γp_t and the value of the firm will be outside the smallest contour. The probability that γp_t and the value of the firm will be outside the next largest contour drops to 1 percent; and the probability that γp_t and the value of the firm will be outside the largest contour is only 1/2 percent. Given the joint density in Figure 3, the likelihood of default in equation (8) is less than 1/4 percent, the volume above the shaded default area in quadrant III and underneath the probability density function.

Figure 3
Firm Value and Asset Prices: The Case of Zero Correlation

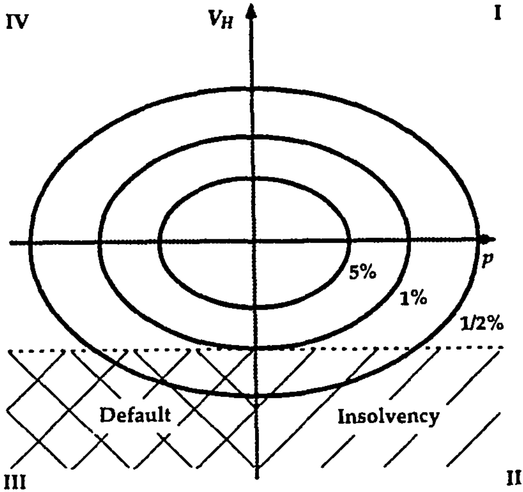
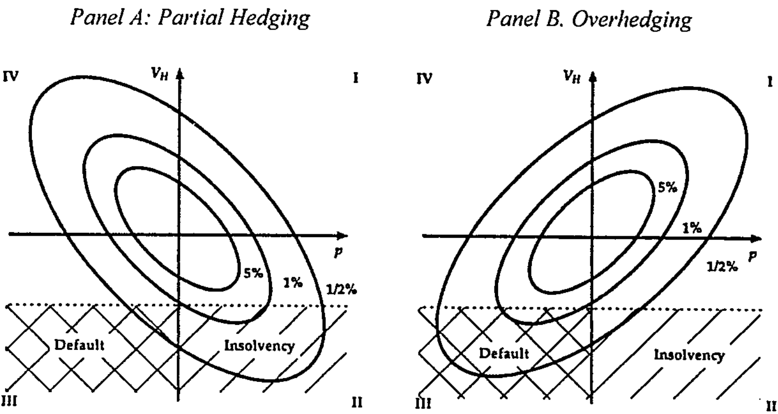


Figure 3 presumes that p_t and the value of the firm are uncorrelated. While this might be the case, in general it is not; derivative payoffs and the value of the firm could have either positive or negative correlation. Figure 4 illustrates how the correlation affects the likelihood of default using a hypothetical example. In both panels of Figure 4, the distribution shows a strong correlation between the value of the firm and the payoff from the derivative. These correlations can be interpreted based on the linear regression of V_H on p_t. This regression has a slope of

$$\Sigma_{pp}^{-1} \operatorname{Cov}(p_t, V_H) = \Sigma_{pp}^{-1} \operatorname{Cov}(p_t, V_0) + \gamma = -\gamma^* + \gamma. \tag{9}$$

For variance-minimizing hedge positions, $\gamma = \gamma^*$, and the hedged firm has zero net exposure to p_t. For all other hedge positions, the firm continues to have residual exposures. In the scalar case, for small hedge positions ($\gamma < \gamma^*$), firm value and p_t are negatively correlated. Conversely, for large hedge positions ($\gamma > \gamma^*$), firm value and p_t are positively correlated.

Figure 4
Default and the Correlation Between Firm and Derivative Value



Partial hedging. Panel A of Figure 4 illustrates the negative correlation between firm value and p_t that is implied by the previously assumed negative correlation between V_H and x_t (Figure 1) and a positive correlation between p_t and x_t. This situation would arise when the firm in Figure 1 is holding fewer than the variance-minimizing number of contracts ($0 < \gamma < \gamma^*$). In this case, a long position in the derivative ($\gamma > 0$) is a hedge for the firm's core exposure, and the likelihood of default on the contract is lower than if firm and derivative value are uncorrelated.[12] The 1 percent confidence region barely touches the shaded area of quadrant III. Compared to Figure 3, a considerable amount of the probability mass has been shifted from quadrant III to quadrant II—away from the default area.[13]

Overhedging. Panel B of Figure 4 illustrates the case of positive correlation between p_t and firm value. This situation can arise if a firm with negative core exposures to p_t acquires a very large long position in the derivative ($\gamma > \gamma^*$). For such a firm, the likelihood of distress-induced default on the derivatives is higher. Now the 1 percent confidence region reaches well into the shaded area of quadrant III, and probability mass has been shifted to the default area. Alternatively, a positive correlation can result if a firm with positive core exposures to p_t also takes a long position in the derivative.

---

[12] This is true in the typical case when there is considerable residual variation. If a perfect hedge were possible (if $|Corr(x_t, p_t)| = 1$), then the perfectly hedged firm would never become insolvent or default.

[13] The panel assumes that the negative net exposure does not stem from a large short position in the derivative. If the firm were to take a short position in p_t with $\gamma < 0$, then the default area would be the area of insolvency in quadrant II.

Figure 4 shows that the likelihood of distress-induced default on derivatives increases with the correlation between the value of the firm and the value of the derivative.

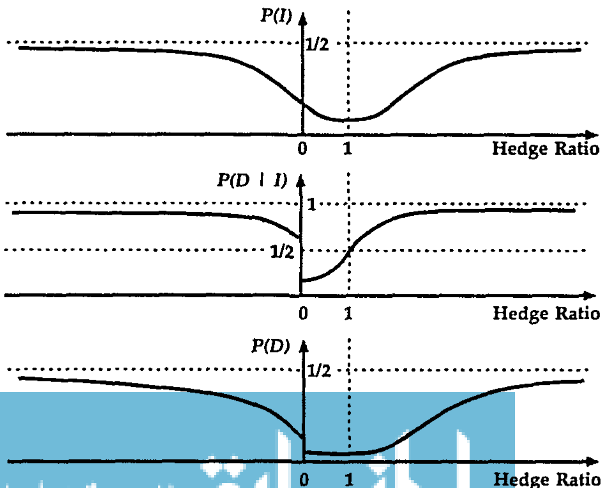
Decomposing the Probability of Default

One can decompose the probability of default on a derivative, P(D), into the probability of insolvency, P(I), and the probability of default conditional on insolvency, P(D | I):

$$P(D) = P(I) \times P(D|I) = P(V_H < V_I) \times P(p_t\gamma < 0|V_H < V_I). \tag{10}$$

Figure 5 presents this decomposition graphically for a range of hedge ratios, γ/γ^*. The probability of insolvency and the probability of default conditional on insolvency both depend on the correlation between the value of the firm and the value of the derivative. This correlation changes as the firm varies its hedge ratio. With incomplete hedging, as the firm increases the size of its hedging position from $\gamma/\gamma^* = 0$, it reduces its net exposure and the volatility of firm value. When the firm fully hedges its exposures ($\gamma/\gamma^* = 1$), the derivatives position minimizes variations in firm value and the risk of insolvency. This is illustrated in the top panel of Figure 5. As the firm increases its hedge ratio beyond this point, the firm over-hedges and reverses its net exposure. Eventually, firm volatility is dominated by the variations in the derivatives position.

Figure 5
Derivatives Positions and the Probability of Default



When the hedge ratio is below zero, the firm is using derivatives to increase its exposures rather than to reduce them. Nonetheless, as long as firm value must fall below its expectation to induce insolvency, the probability of insolvency is less than 1/2. This is indicated by the fact that, regardless of correlation, only half of the probability mass is below the horizontal axis in Figure 4.

Figure 5 shows that firms that hedge their exposures with derivatives are safer loan customers than firms that do not hedge their exposures. Nonetheless, firms that use derivatives to amplify their inherent exposures are more likely to become insolvent and hence are worse credit risks.

The middle panel of Figure 5 shows how the probability of default on a derivative conditional on insolvency depends on the hedge ratio. As the firm increases its hedge ratio above zero, it also increases the correlation between the value of the firm and the value of the derivatives. This consequently increases the probability of default conditional on insolvency, since more of the probability mass is shifted into the default region. At a hedge ratio of one, firm and derivative value are uncorrelated (as in Figure 3) and the probability of default given insolvency is 1/2 for symmetric unexpected changes. But, if the hedge ratio is negative, default risk jumps immediately. In Panel A of Figure 4, this would have the effect of switching the default area into quadrant II; hence, the discontinuous increase in the default probability.[14] In the extremes, if the firm acquires very large derivatives positions, the firm is sure to default on these positions in the event of insolvency.

Alternatively, the probability of default conditional on insolvency can be interpreted as the probability of default on derivatives relative to the probability of default on debt, $P(D|I) = P(D)/P(I)$. The middle panel of Figure 5 shows that the probability of default on derivatives is always less than the probability of default on debt. Furthermore, for derivatives used to hedge ($0 < \gamma/\gamma^* < 1$), default on the derivatives is never more than half as likely as default on debt. Not only is the default risk of derivatives significantly lower than that of the firm's debt, but hedging with derivatives helps reduce the default risk of debt by offsetting the firm's core business exposures. Without doubt, the probability of default on derivatives that are used to hedge is low by the standards of default on corporate debt.

Finally, the bottom panel of Figure 5 explicitly shows the probability of default on the derivative—the product of the probabilities in the two panels above. The probability of default is always less than 1/2, and much lower than that for typical derivative positions with hedge ratios between zero and one. Nonetheless, as the size of the derivatives position becomes very large, net firm value (including the derivatives) and the derivatives become more highly correlated. For extreme positions, the firm's payoffs are almost entirely derivatives related. If the derivative payoffs are symmetrically distributed about zero, there is a 50 percent chance that the derivative finishes out of the money, that the firm becomes insolvent, and defaults on the derivatives contracts.

---

[14] If (unhedged) firm value and the derivative are uncorrelated, then the probabilities are symmetric about zero derivatives positions and do not have a discontinuity at $\gamma = 0$.

Therefore, insurers that act as market makers for derivatives have an interest in knowing whether the derivatives are used for hedging or speculation. Derivatives used for hedging ($0 \leq \gamma/\gamma^* \leq 1$) present much smaller default risks than derivatives used to "double up" an exposure ($\gamma/\gamma^* < 0$). Of course, the level of concern will be much less for a firm with high credit standing, since the counterparty's equity would have to be exhausted before the market maker suffered losses.

To the extent that corporations use derivatives to hedge, we can use general corporate default rates to assess the default risk of derivatives. Altman (1989) reports that 0.93 percent of all A-rated corporate bonds default during the first ten years after being issued. This evidence suggests an average annual default rate of 0.1 percent. For the special case of a firm negotiating an at-market swap that completely hedges the firm's interest rate exposure, the default rate on the swap will be half the default rate on the debt. (For an at-market swap, future interest rates are as likely to be above as below the swap rate; if the firm completely hedges its exposure to interest rates, firm value and interest rates are uncorrelated.) Consequently, a conservative estimate of the average annual default rate on swaps used to hedge the exposures of an A-rated firm is 1/20 of one percent (see Figure 5).

Default Exposure

In the special case of independence between the derivatives payoffs and firm value, the default exposure is simply the product of the expected exposure and the probability of default. We have already reported estimates of each of these two components. Thus, our conservative estimate of the default probability is 0.0005. We have also argued that the expected loss on an unsecured swap is 0.5 percent of notional principal; this reflects a mark-to-market value of 1 percent of notional principal and expected recovery rate of 50 percent for unsecured financial claims in default. Therefore, a conservative estimate of the annual expected default cost is 0.00025 percent of notional principal. This means that, on a $10 million interest rate swap, the expected annual cost of default is no more than $25.[15]

Corporate Use of Derivatives

At this point, the evidence on corporate derivative use is still somewhat preliminary. Yet, the existing evidence supports the hypothesis that firms use derivatives to hedge.

Dolde (1993) reports the results of a survey of the risk-management practices of 244 Fortune 500 firms. The overwhelming majority responded that their policy is to hedge with hedge ratios between zero and one. Although many firms adjust this hedge ratio on the basis of their market view, only two of the 244 firms responded that they sometimes choose hedge ratios outside the zero to one range. Furthermore, a considerable body of theory (see Mayers and Smith, 1982, 1987;

---

[15] For longer-maturity derivative contracts, this annual default cost must be scaled up when considering the default cost over the life of the contract.

Main, 1983; Stulz, 1984; Smith and Stulz, 1985; and Froot, Scharfstein, and Stein, 1993) predicts a set of firm characteristics that should be associated with higher demand for hedging and hence larger derivatives positions. Nance, Smith, and Smithson (1993); Booth, Smith, and Stolz (1984); Block and Gallagher (1986); Houston and Mueller (1988); Wall and Pringle (1989); Mian (1996); Tufano (1996); Geczy, Minton, and Schrand (1997); and Hentschel and Kothari (1997) generally report empirical support for these predictions. Yet, if firms were using derivatives simply to speculate, one would not expect to observe this association between firm characteristics and derivatives use.

Although the overall evidence suggests that firms typically use derivatives to hedge, we believe it is important to note that derivatives dealers have incentives to monitor customers' use of derivatives to ensure that they use derivatives to hedge.

SYSTEMIC RISK FROM DERIVATIVES

As noted above, one of the prominent concerns of regulators is "systemic risk" arising from derivatives. Although this risk is rarely defined and almost never quantified, the systemic risk associated with derivatives is often envisioned as a potential domino effect in which default in one derivative contract spreads to other contracts and markets, ultimately threatening the entire financial system.

For the purposes of this article, we define the systemic risk of derivatives as widespread default in any set of financial contracts associated with default in derivatives. If derivative contracts are to cause widespread default in other markets, there first must be large defaults in derivative markets. In other words, significant derivative defaults are a necessary (but not sufficient) condition for systemic problems. While this interpretation of systemic risk is consistent with most others, we believe that focusing on default is useful because it has definite cash-flow consequences and is more operational.[16]

Even if systemic risk is simply the aggregation of the underlying risks, because the underlying risks are correlated, one cannot just sum them to find the total. Some argue that widespread corporate risk management with derivatives increases the correlation of default among financial contracts. Yet this argument fails to recognize that the adverse effects of shocks on individual firms should be smaller precisely because the same shocks are spread more widely. More important, to the extent that firms use derivatives to hedge their existing exposures, much of the impact of shocks is being transferred from corporations and investors less able to bear them to counterparties better able to absorb them.

It is conceivable that financial markets could be hit by a very large disturbance. The effects of such a disturbance on derivative markets and participants in these markets depend, in particular, on the duration of the disturbances and whether firms suffer common or independent shocks.

---

[16] The Bank for International Settlements (1992), for example, defines systemic risk to include "widespread difficulties." Although this definition agrees with ours in spirit, it is less operational.

If the disturbance were large but temporary, many outstanding derivatives would be essentially unaffected because they specify only relatively infrequent payments.[17] Therefore, a temporary disturbance would primarily affect contracts with required settlements during this period.

If the shock were permanent, it would affect derivatives in much the same manner that it affects other instruments. If the underlying price increases, long positions gain while short positions lose. Since derivative contracts are in zero net supply, the gains exactly equal the losses.

A critical question in evaluating systemic risk concerns the extent to which defaults across derivatives markets, and financial markets in general, are likely to be correlated. There are reasons to expect that defaults on derivatives contracts are approximately independent across dealers and over time. Dealers have powerful incentives to assess the default risks of their customers. In practice, a strong credit rating is required of derivatives customers. Second, as we have discussed in detail, firms using derivatives to hedge their exposures are most likely to become insolvent precisely when their derivatives are in the money. Price shocks in the underlying derivative do not cause these firms to default on the derivatives.

In this sense, derivative defaults are significantly more idiosyncratic than defaults on loans. For example, a large increase in interest rates is much more likely to produce defaults on floating-rate bank loans than on interest rate swaps. For partially-hedged firms, the correlation among loan defaults is likely to be higher than the correlation among derivative defaults. Hence, diversification is a more effective tool for managing the credit risk of derivatives than loans. This is why derivatives dealers carefully monitor and ultimately limit their exposures to individual counterparties, industries, and geographical areas.

Finally, dealers with a carefully balanced book and substantial capital reserves can absorb individual defaults by their counterparties without defaulting on their other outstanding contracts.

AGENCY RISK

The derivatives losses incurred by firms like Procter & Gamble, Gibson Greetings, and Barings Bank gained notoriety because of their size—not because there was serious concern that the companies would default on the contracts. Nevertheless, these losses share a disturbing pattern of inappropriate incentives and ineffective controls within the firms. In many instances, the magnitudes of the derivative losses and, hence, the underlying derivative positions came as surprises to senior management and shareholders. This suggest that employees with the authority to take such derivatives positions were acting outside their authorized scope and were not acting in the best interests of the firms' owners.

The misalignment between owners' objectives and employees' actions makes this a typical agency problem. Employees in the derivatives area (the agents) are

---

[17] The payments under path-dependent derivatives—like Asian options, for example—can be affected by temporary disturbances long after the disturbance has subsided.

not working toward the general corporate objectives set by senior management and shareholders (the principals). Given the nature of the problem, we will refer to the associated risks as "agency risks."

In our simple setting, such agency problems exist whenever the agent in charge of determining the actual derivatives position, γ, has private incentives to deviate from the position that maximizes the value of the firm. For concreteness, we assume that this optimal position is γ^*, as it would be in the absence of hedging costs.

Problems of this type are not specific to derivatives; they arise in many different settings where principals and agents have divergent interests. Since the agent's incentives are affected by the structure of the organization, the design of the organization can either exacerbate or control these incentive problems. There are three critical facets of organizational structure: evaluation and control systems, compensation and reward systems, and assignments of decision rights (see Brickley, Smith, and Zimmerman, 1996). Although no single organizational structure is appropriate for all firms, there are several general features that should help control agency risk in derivatives.

Evaluation and Control

Agency problems would be minimal if both the core exposure, $-\gamma^*$, and the actual derivatives position, γ, were perfectly observable. In this case, firm owners would simply enforce $\gamma = \gamma^*$ by rewarding compliance and punishing deviations.

Recent regulatory proposals to increase the required disclosure of firms' derivatives positions can be interpreted as an attempt to make γ better observable and, hence, reduce monitoring costs.

But neither current nor proposed disclosure standards achieve perfect observability. For one reason, firms are required to report only the size of their derivative positions; they do not have to provide information about the exposures ("deltas" in options parlance) of these positions. In addition, the exposures are only reported on the balance-sheet dates, yet exposures can change over time even if the positions remain fixed. Moreover, for outsiders, the core exposures of the firm can be difficult to measure, making it hard to ascertain whether the firm is engaged in an optimal hedging program.

Our simple model abstracts from information costs. If it is costly to gather and maintain this information, or if the information has private value, full disclosure is generally suboptimal. Nonetheless, net, core, and derivatives exposures are surely among the most important pieces of summary information in evaluating the performance of a firm's derivatives activity.

The recent derivatives scandals also point out, however, that monitoring even within the firm can be difficult. Managers at firms like Barings Bank and Procter & Gamble claim they were not aware of the full extent of the derivatives activities of their subordinates. In a less dramatic incident, an internal reorganization at American International Group apparently responded to lapses that occurred because a single individual had run and evaluated the firm's main derivatives activities. This is an internal control problem that financial accounting standards simply cannot solve. Our analysis summarized in Figure 5 provides some solace in this regard. The actual hedge position must deviate materially from the variance-minimizing position to significantly increase the risk of insolvency or default on the derivatives. Nonetheless, even in the absence of default, such deviations have costs that are determined by the curvature of firm value.

Careful control and supervision is critically important for derivatives. Although leverage is one of the features that makes derivatives attractive hedging instruments, it also makes it harder to monitor derivatives activity by reducing the cash flows at initiation of the contracts. The problem can be compounded by the steady increase in available maturities, which extend the time required to determine the ultimate net gain or loss from the contracts.

Compensation and Incentives

Even if the actual derivatives activity, γ, is costly to observe, compensation based on the outcome, V_H, may still induce the desired behavior. By tying compensation to the objective, one can induce employees in the derivatives area to adhere to the hedging program if the firm's core exposures are observable. Even if γ cannot be observed directly, if it can be inferred from V_H on the basis of x_t, p_t, and γ^*, incentive compensation can be used to ensure appropriate hedging activities by employees.

Indeed, because the objective is to stabilize firm value or taxable income, incentive compensation for treasury employees might be cheaper to implement than for many other employees, all else equal. The typical cost of incentive compensation is the increased income risk for the employee. Risk-averse employees demand higher average compensation to bear this risk. Yet, employees charged with reducing risk face less risk when they are successful.

For derivatives employees in trading or market-making functions, the situation is somewhat different. Here, the objective is not to stabilize firm value but to generate profits. For any high-leverage financial contract, strong incentive compensation based on the payoff to the contract can have undesirable side effects.

The primary problem in linking pay to derivative profits is the limited liability of employees. Although employees can participate in the upside, they usually have insufficient resources to share large negative outcomes. This asymmetry induces option-like features in compensation plans based on trading profits. Compensating employees on the basis of long-term performance reduces these option-like features that would otherwise encourage traders to take riskier positions than is optimal from the owners' perspective.

One way to reward traders for good performance without forgiving all losses is to base more of the compensation on long-term performance. For example, in a good year, a trader might have part of a bonus paid into a deferred compensation account. If subsequent performance is also good, the account continues to grow. On the other hand, if the trader is simply taking large bets, half of which lose, then the bonus account is reduced during years with poor performance. In this way, derivatives traders share responsibility for their losses as well as gains.

Decision Rights

For most corporations, derivatives activity is not entirely static, the way we have characterized it in our model, but it moves relatively slowly. Most firms' hedging demands do not change much on a daily basis. In such cases, it is not critical that individual employees have decision rights over derivatives positions. Allocating the decision rights to a team of treasury employees is likely to improve internal controls at low cost.

In contrast, derivatives traders and dealers typically have—and should have—substantial decision rights over the positions they assume in derivatives. Moreover, typical employment in the derivatives area also suggests that an optimal compensation package should have strong incentive components (see Holmstrom, 1979). In particular, improved performance can generate very large additional profits, and normal trading activities are readily observable. There is also no reason to believe that employees in the derivatives area are more risk averse or less responsive to incentives than other employees.

Setting position limits for traders constrains the size of the positions that they could assume. Separating trading and settlement responsibilities (something that apparently was not done in the case of Barings Bank) allows firms to monitor derivatives activities. This separation is also necessary to ensure compliance with position limits.

Many firms are changing the ways in which they manage their derivatives operations to account for these agency issues. As we gather more experience with these compensation and control systems, control of these problems is likely to improve. Nevertheless, the recent losses demonstrate that agency risk is currently a material problem for many firms.

CONCLUSION

We provide a parametric model of hedging which captures the major hedging theories. With the aid of this model, we show that firms that use derivatives have lower default probabilities on these derivatives than they do on their debt. Based on this insight and empirical evidence on bond default rates, we compute a conservative default probability for derivatives. We estimate that the expected annual loss due to default on a $10 million interest rate swap is unlikely to exceed $25.

Given these small default rates, we argue that systemic risk, the probability of widespread default, is even smaller. To the extent that derivatives are being used primarily to hedge rather than to speculate, the default probability associated with derivatives is less than half the default probability on debt issued by the same firms. Furthermore, derivatives markets act to reduce systemic risk by spreading the impact of underlying economic shocks among a larger set of investors in a better position to absorb them.

Establishing effective public policy toward derivatives requires accurate assessment of both the risks associated with derivatives and the benefits offered by the instruments. This balanced assessment seems to be missing in many current derivatives rules. For example, the California Insurance Commissioner recently circulated a bulletin that restated the policy on derivatives. While these rules allow insurers to use some derivatives, California law apparently requires "explicit, legal authorization" for new investment devices, including derivatives. Moreover, the commissioner's interpretation that "what existing law doesn't authorize is consequently unauthorized" reflects a widespread but one-sided view of derivatives. Of course, the misuse of derivatives can be costly. Nevertheless, a growing body of academic evidence suggests that these tools are typically used by firms to hedge their exposures and that such hedging can provide material benefits to these firms.

Although we conclude that default and systemic risks are not major problems in derivatives markets, we argue that many firms are exposed to agency risk. This risk arises when employees have decision rights over derivatives and misaligned incentives relative to the firm but are not properly monitored. The proper balancing of decision rights, incentives, and control is a major firm-internal concern for firms with derivatives activity.

We are concerned about this misidentification of the nature of the risk that regulation might address. Although many regulatory proposals focus on default and systemic risk, the problem cases appear to involve agency risk. The internal nature of this problem is apparently not recognized in many regulatory proposals, nor is regulation likely to be a particularly effective tool in overcoming this problem. Nonetheless, recent disclosure proposals which would allow firms to use private valuation models for their derivatives positions form a noticeable exception. Such standards encourage monitoring at least as much as they encourage transparent disclosure.

REFERENCES

Adler, Michael and Bernard Dumas, 1984, Exposure to Currency Risk: Definition and Measurement., *Financial Management*, 13: 41–50.

Altman, Edward I., 1989, Measuring Corporate Bond Mortality and Performance, *Journal of Finance*, 44: 909–922.

Bank for International Settlements, 1992, *Recent Developments in International Interbank Relations* (Basle: Bank for International Settlemtents).

Barclay, Michael J. and Clifford W. Smith, Jr., 1995, The Maturity Structure of Corporate Debt, *Journal of Finance*, 50: 609–631.

Black, Fischer and Myron S. Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81: 637–654.

Block, S. and T. J. Gallagher, 1986, The Use of Interest Rate Futures and Options by Corporate Financial Managers, *Financial Management*, 15: 73–78.

Booth, James R., Richard L. Smith, and Richard W. Stolz, 1984, Use of Interest Rate Futures by Financial Institutions, *Journal of Bank Research*, 15: 15–20.

Brickley, James A., Clifford W. Smith, Jr., and Jerold L. Zimmerman, 1996, *Organizational Architecture: A Managerial Economics Approach* (Burr Ridge, Ill.: Irwin).

Cooper, Ian A. and Antonio S. Mello, 1991, The Default Risk of Swaps, *Journal of Finance*, 46: 597–620.

Dolde, Walter, 1993, The Trajectory of Corporate Financial Risk Management, *Journal of Applied Corporate Finance*, 6: 33–41.
Flannery, Mark J. and Christopher M. James, 1984, The Effect of Interest Rate Changes on the Common Stock Returns of Financial Institutions, *Journal of Finance*, 39: 1141–1153.
Franks, Julian R. and Walter N. Torous, 1994, A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations, *Journal of Financial Economics*, 35: 349–370.
Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein, 1993, Risk Management: Coordinating Corporate Investment and Financing Policies, *Journal of Finance*, 48: 415–427.
Geczy, Christopher, Bernadette A. Minton, and Catherine M. Schrand, 1997, Why Firms Hedge: Distinguishing Among Existing Theories, *Journal of Finance*, forthcoming.
Hentschel, Ludger and S. P. Kothari, 1997, Are Corporations Reducing or Taking Risks with Derivatives? Unpublished Paper, William E. Simon Graduate School of Business Administration, University of Rochester, Rochester, New York.
Hentschel, Ludger and Clifford W. Smith, Jr., 1995, Controlling Risks in Derivatives Markets, *Journal of Financial Engineering*, 4: 101–125.
Holmstrom, Bengt, 1979, Moral Hazard and Observability, *Bell Journal of Economics*, 10: 74–91.
Houston, C. O. and G. G. Mueller, 1988, Foreign Exchange Rate Hedging and SFAS No. 52—Relatives or Strangers? *Accounting Horizons*, 2: 50–57.
Jarrow, Robert A. and Stuart M Turnbull, 1995, Pricing Derivatives on Financial Securities Subject to Credit Risk, *Journal of Finance*, 50: 53–85.
Johnson, Herbert E. and René Stulz, 1987, The Pricing of Options with Default Risk, *Journal of Finance*, 42: 267–280.
Longstaff, Francis A. and Eduardo S. Schwartz, 1995, A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, *Journal of Finance*, 50: 789–819.
Main, B. G. M., 1983, Corporate Insurance Purchases and Taxes, *Journal of Risk and Insurance*, 50: 197–223.
Mayers, David and Clifford W. Smith, Jr., 1982, On the Corporate Demand for Insurance, *Journal of Business*, 55: 281–296.
Mayers, David and Clifford W. Smith, Jr., 1987, Corporate Insurance and the Underinvestment Problem, *Journal of Risk and Insurance*, 54: 45–54.
Merton, Robert C., 1973, Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science*, 1: 141–183.
Mian, Shezad L., 1996, Evidence on Corporate Hedging Policy, *Journal of Financial and Quantitative Analysis*, 31: 419–439.
Nance, Deana R., Clifford W. Smith, Jr., and Charles W. Smithson, 1993, On the Determinants of Corporate Hedging, *Journal of Finance*, 48: 267–284.
Smith, Clifford W., Jr., and René M. Stulz, 1985, The Determinants of Firms' Hedging Policies, *Journal of Financial and Quantitative Analysis*, 20: 391–405.
Sorensen, Eric H. and Thierry F. Bollier, 1994, Pricing Swap Default Risk, *Financial Analysts Journal*, 50: 23–33.
Stulz, René M., 1984, Optimal Hedging Policies, *Journal of Financial and Quantitative Analysis*, 19: 127–140.
Sun, Tong-sheng, Suresh Sundaresan, and Ching Wang, 1993, Interest Rate Swaps, An Empirical Investigation, *Journal of Financial Economics*, 34: 77–99.

Tufano, Peter, 1996, Who Manages Risk? An Empirical Examination of Risk Management Practices in the Gold Mining Industry, *Journal of Finance*, 51: 1097–1137.

Wall, Larry D. and John J. Pringle, 1988, Interest Rate Swaps: A Review of the Issues, *Economic Review* (Federal Reserve Bank of Atlanta), November/December: 22–40.

Wall, Larry D. and John J. Pringle, 1989, Alternative Explanations of Interest Rate Swaps: An Empirical Investigation, *Financial Management*, 18: 59–73.